APPENDIX D

THREE-SLAB HEAT FLOW DERIVATION

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In principle the heat flow equation, $u_t = ku_{xx}$, can be solved for any number of slabs for any initial and boundary conditions (Carlslaw and Jaeger, 1959). But the solution for a particular set of initial and boundary conditions and number of slabs involves considerable labor.[3] The problem to be solved was for three slabs, a single slab of one material surrounded by identical slabs of a second material (Fig. D.1). From symmetry $u(x,t) = u(-x,t)$. Initial and boundary conditions are $u(x,a) = T_1$ for $x \epsilon (o,a)$, $u(x,o) = T_2$ for $x \epsilon (a,b)$ and $u(b,t) = 0$. At slab interfaces, jump conditions of continuity of temperature and heat flux apply.

The problem can be solved using the Laplace transformation. For regions numbered as in Fig. D.1, the problem is as follows:

$$\text{Region 1,} \quad \varphi_t = k\varphi_{xx}$$

$$\text{Region 2,} \quad \Psi_t = \kappa\Psi_{xx}$$

$$\text{Region 3,} \quad \eta_T = \kappa\eta_{xx}$$

Initial conditions

$$\varphi(x,o) = T_1 \quad x \epsilon (-a,a)$$

$$\Psi(x,o) = T_2 \quad x \epsilon (a,b)$$

$$\eta(x,o) = T_2 \quad x \epsilon (-b,-a)$$

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[3]The solution to the present problem was obtained in cooperation with G. Swan, Department of Applied Mathematics, Washington State University.